Blame Assignment for Higher-Order Contracts with Intersection and Union

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Higher-Order Contracts

- $Even = flat(\lambda x.x\%2 = 0)$
- $Odd = flat(\lambda x.x\%2 = 1)$

Assertion ($Even \rightarrow Even$)

Let $add2Even = ((\lambda x.x + 2) \text{ @ } (Even \rightarrow Even))$

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- $(add2Even\ 2) \longrightarrow^{*} 4$ ✓

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- $(add2Even\ 1) \longrightarrow^{*}$ ✗ blame context □ 1

Assertion ($Odd \rightarrow Odd$)

Let $add2Odd = ((\lambda x.x + 2)$ @ $(Odd \rightarrow Odd))$

- $\mathit{Even} = \mathit{flat}(\lambda x.x\%2 = 0)$
- $\mathit{Odd} = \mathit{flat}(\lambda x.x\%2 = 1)$

Assertion $(\mathit{Even} \rightarrow \mathit{Even})$

Let $\mathit{add2Even} = ((\lambda x.x + 2)\ @\ (\mathit{Even} \rightarrow \mathit{Even}))$

- $(\mathit{add2Even}\ 2) \longrightarrow^* 4$ ✓
- $(\mathit{add2Even}\ 1) \longrightarrow^*$ ✗ blame context $\square$ 1

Assertion $(\mathit{Odd} \rightarrow \mathit{Odd})$

Let $\mathit{add2Odd} = ((\lambda x.x + 2)\ @\ (\mathit{Odd} \rightarrow \mathit{Odd}))$

- $(\mathit{add2Odd}\ 1) \longrightarrow^* 3$ ✓

Higher-Order Contracts

- $Even = flat(\lambda x.x\%2 = 0)$
- $Odd = flat(\lambda x.x\%2 = 1)$

Assertion ($Even \rightarrow Even$)

Let $add2Even = ((\lambda x.x + 2)$ @ $(Even \rightarrow Even))$

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- $(add2Even\ 1) \longrightarrow^*$ ✗ blame context $\square$ 1

Assertion ($Odd \rightarrow Odd$)

Let $add2Odd = ((\lambda x.x + 2)$ @ $(Odd \rightarrow Odd))$

- $(add2Odd\ 1) \longrightarrow^* 3$ ✓
- $(add2Odd\ 2) \longrightarrow^*$ ✗ blame context $\square$ 2

Combination of Contracts

Observation

- $\lambda x.x + 2$ works for *even* and *odd* arguments

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- $\lambda x.x + 2$ fulfills $\mathit{Odd} \rightarrow \mathit{Odd}$ **and** $\mathit{Even} \rightarrow \mathit{Even}$

Combination of Contracts

Observation

- $\lambda x.x + 2$ works for *even* and *odd* arguments
- $\lambda x.x + 2$ fulfills $Odd \rightarrow Odd$ **and** $Even \rightarrow Even$
- How can we express that with a single contract?

Observation

- $\lambda x.x + 2$ works for *even* and *odd* arguments
- $\lambda x.x + 2$ fulfills $\mathit{Odd} \rightarrow \mathit{Odd}$ **and** $\mathit{Even} \rightarrow \mathit{Even}$
- How can we express that with a single contract?

Intersection Contract!

Inspiration

Intersection Type

- $V : S \cap T$
- Models overloading
- Models multiple inheritances

Union Type

- $V : S \cup T$
- Dual of intersection type
- Domain of overloaded functions

This Work

- Extend higher-order contracts with intersection and union
- Specification based on the type theoretic construction

Assertion $(Even \rightarrow Even) \cap (Odd \rightarrow Odd)$

Let $add2 = ((\lambda x.x + 2)\ @\ (Even \rightarrow Even) \cap (Odd \rightarrow Odd))$

- $(add2\ 2) \longrightarrow^* 4$ ✓
- $(add2\ 1) \longrightarrow^* 3$ ✓

No blame because of the intersection contract!

Flat Contract

- $Even = flat(\lambda x.x\%2 = 0)$
- $Odd = flat(\lambda x.x\%2 = 1)$
- $Pos = flat(\lambda x.x > 0)$

Examples

- $Pos \cap Even$

Flat Contract

- $flat(\lambda x.P) \cap flat(\lambda x.Q) \equiv flat(\lambda x.P \wedge Q)$

Intersection Contract

Assertion

Let $add1 = ((\lambda x.x + 1) \mathbin{@} (Even \to Even) \cap (Pos \to Pos))$

Definition

- Context gets <u>*blamed*</u> for $\mathcal{C} \cap \mathcal{D}$ iff:
  (Context gets *blamed* for $\mathcal{C}$) $\wedge$ (Context gets *blamed* for $\mathcal{D}$)
- Subject M gets <u>*blamed*</u> for $\mathcal{C} \cap \mathcal{D}$ iff:
  (M gets *blamed* for $\mathcal{C}$) $\vee$ (M gets *blamed* for $\mathcal{D}$)

Intersection Contract

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Let $add1 = ((\lambda x.x + 1) \;@\; (Even \rightarrow Even) \cap (Pos \rightarrow Pos))$

- $(add1\ 3) \quad \longrightarrow^{*} 4$ ✓

Definition

- Context gets *blamed* for $\mathcal{C} \cap \mathcal{D}$ iff:
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Intersection Contract

Assertion

Let $add1 = ((\lambda x.x + 1) \ @ \ ({\color{red}Even} \to Even) \cap ({\color{red}Pos} \to Pos))$

- $(add1\ 3) \longrightarrow^* 4$ ✓
- $(add1\ -1) \longrightarrow^*$ ✗ blame context $\square$ -1

Definition

- Context gets *blamed* for $\mathcal{C} \cap \mathcal{D}$ iff:
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Intersection Contract

Assertion

Let $add1 = ((\lambda x.x + 1)\ @\ (Even \rightarrow Even) \cap (Pos \rightarrow Pos))$

- $(add1\ 3) \longrightarrow^* 4$ ✓
- $(add1\ -1) \longrightarrow^*$ ✗ blame context $\Box\ -1$
- $(add1\ 2) \longrightarrow^*$ ✗ blame subject $(\lambda x.x + 1)$

Definition

- Context gets *blamed* for $\mathcal{C} \cap \mathcal{D}$ iff:
  (Context gets *blamed* for $\mathcal{C}$) $\wedge$ (Context gets *blamed* for $\mathcal{D}$)
- Subject M gets *blamed* for $\mathcal{C} \cap \mathcal{D}$ iff:
  (M gets *blamed* for $\mathcal{C}$) $\vee$ (M gets *blamed* for $\mathcal{D}$)

Contract Assertion

Example

- $((\lambda x.x + 1) \mathbin{@} (\mathit{Even} \rightarrow \mathit{Even}) \cap (\mathit{Pos} \rightarrow \mathit{Pos}))\ 3 \longrightarrow^* 4$

- A failing contract must not signal a violation immediately
- Violation depends on combinations of failures in different sub-contracts
- Contract assertion must connect each contract with the enclosing operations

Contract Assertion

Example

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Operational Semantics

Reduction Relation

$$\varsigma, M \longrightarrow \varsigma', N$$

- M, N expressions
- ς list of constraints
- One constraint for each contract operator $\rightarrow$, $\cap$, $\cup$
- One constraint for each flat contract
- Blame calculation from a list of constraints

Flat Contract

Evaluation Rule

Flat

$$\frac{M\ V \longrightarrow^{*} W \qquad \varsigma' = \flat\blacktriangleleft(W) : \varsigma}{\varsigma, E[V \mathbin{@^{\flat}} \mathit{flat}(M)] \longrightarrow \varsigma', E[V]}$$

Interpretation

Interpretation of a constraint list

$$\mu \in ((\!|\flat|\!) \times \{\mathit{subject}, \mathit{context}\}) \to \mathbb{B}$$

- An interpretation μ is a mapping from blame label $\flat$ to records of elements of $\mathbb{B} = \{\mathsf{t}, \mathsf{f}\}$, order $\mathsf{t} \sqsubset \mathsf{f}$
- Ordering reflects gathering of information with each execution step
- Each blame label $\flat$ is associated with two truth values, $\flat.\mathit{subject}$ and $\flat.\mathit{context}$

Flat Contract (cont'd)

Evaluation Rule

$$\frac{\textsc{Flat}\quad M\ V \longrightarrow^{*} W \qquad \varsigma' = \flat\blacktriangleleft(W) : \varsigma}{\varsigma, E[V \mathbin{@^{\flat}} \mathit{flat}(M)] \longrightarrow \varsigma', E[V]}$$

Constraint Satisfaction

$$\frac{\textsc{C-Flat}\quad \mu(\flat.\mathit{subject}) \sqsupseteq W \qquad \mu(\flat.\mathit{context}) \sqsupseteq \mathsf{t}}{\mu \models \flat \blacktriangleleft W}$$

Blame Calculation

Definition

ς is a *blame state* if there exists a top-level blame label such that

$$\mu(\flat.\mathit{subject}) \sqsupseteq \mathsf{f} \;\vee\; \mu(\flat.\mathit{context}) \sqsupseteq \mathsf{f}$$

- Evaluation stops if a blame state is reached.

Function Contract

Evaluation Rule

FUNCTION

$$\frac{\flat_1, \flat_2 \notin \varsigma \qquad \varsigma' = \flat \blacktriangleleft (\flat_1 \rightarrow \flat_2) : \varsigma}{\varsigma, E[(V \mathbin{@^{\flat}} (\mathcal{C} \rightarrow \mathcal{D}))\ W] \longrightarrow \varsigma', E[(V\ (W \mathbin{@^{\flat_1}} \mathcal{C})) \mathbin{@^{\flat_2}} \mathcal{D}]}$$

Constraint Satisfaction

C-FUNCTION

$$\frac{\mu(\flat.\mathit{subject}) \sqsupseteq \mu(\flat_1.\mathit{context} \wedge (\flat_1.\mathit{subject} \Rightarrow \flat_2.\mathit{subject})) \qquad \mu(\flat.\mathit{context}) \sqsupseteq \mu(\flat_1.\mathit{subject} \wedge \flat_2.\mathit{context})}{\mu \models \flat \blacktriangleleft \flat_1 \rightarrow \flat_2}$$

Intersection Contract

Evaluation Rule

INTERSECTION

$$\frac{\flat_1, \flat_2 \notin \varsigma \qquad \varsigma' = \flat \blacktriangleleft (\flat_1 \cap \flat_2) : \varsigma}{\varsigma, E[(V \mathbin{@^{\flat}} (Q \cap R))\ W] \longrightarrow \varsigma', E[((V \mathbin{@^{\flat_1}} Q) \mathbin{@^{\flat_2}} R)\ W]}$$

Constraint Satisfaction

C-INTERSECTION

$$\frac{\mu(\flat.subject) \sqsupseteq \mu(\flat_1.subject \wedge \flat_2.subject) \quad \mu(\flat.context) \sqsupseteq \mu(\flat_1.context \vee \flat_2.context)}{\mu \models \flat \blacktriangleleft \flat_1 \cap \flat_2}$$

Union Contract

- Dual of intersection contract
- Exchange $\wedge$ and $\vee$ in the blame calculation
- *Delayed* evaluation changes to an *immediate* evaluation

In the Paper

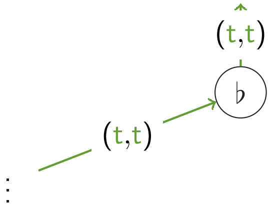
Technical Results

- Contract Rewriting
- Deterministic and nondeterministic specification of contract monitoring
- Denotational specification of the semantics of contracts
- Theorems for contract and blame soundness

Conclusion

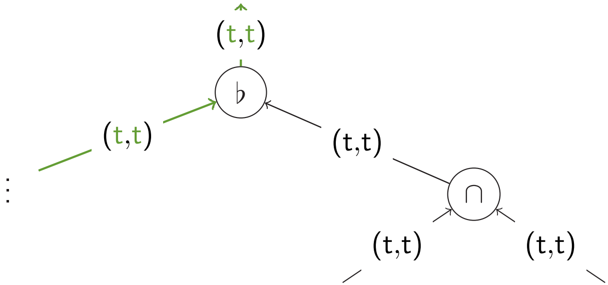
- Intersection and union contracts provide dynamic guarantees equivalent to their type-theoretic counterparts
- Constraint-based blame calculation enables higher-order contracts with unrestricted intersection and union
- Formal basis of *TreatJS*, a language embedded, higher-order contract system implemented for JavaScript

Constraint Graph



Reduction

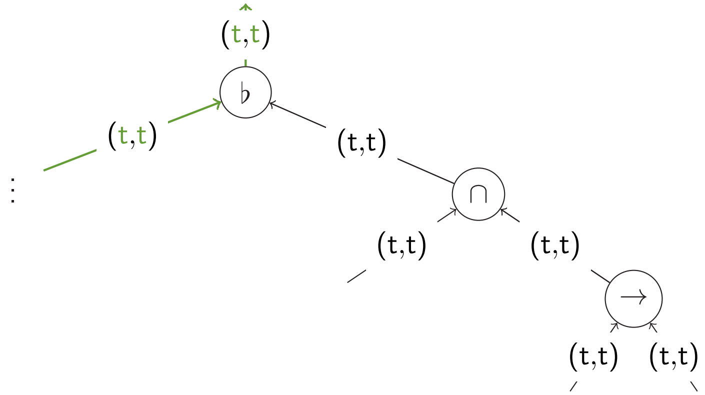
- ς,
  $((\lambda x.x + 1)\ @^\flat\ ((\mathit{Even} \to \mathit{Even}) \cap (\mathit{Pos} \to \mathit{Pos})))\ 0$



Reduction

- $\longrightarrow \flat \blacktriangleleft (\flat_1 \cap \flat_2) : \cdots,$
  $(((\lambda x.x + 1) \ @^{\flat_1} \ (\mathit{Even} \to \mathit{Even})) \ @^{\flat_2} \ (\mathit{Pos} \to \mathit{Pos})) \ 0$

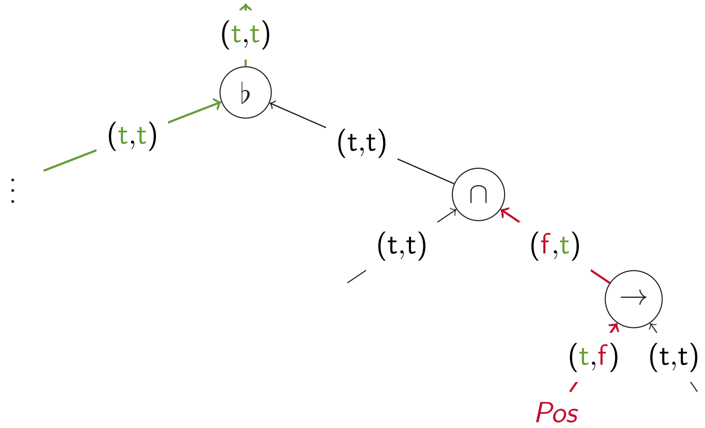
Constraint Graph



Reduction

- $\longrightarrow \flat_2 \blacktriangleleft (\flat_3 \to \flat_4) : \cdots,$
  $(((\lambda x.x + 1) @^{\flat_1} (\mathit{Even} \to \mathit{Even})) (0 @^{\flat_3} \mathit{Pos})) @^{\flat_4} \mathit{Pos}$

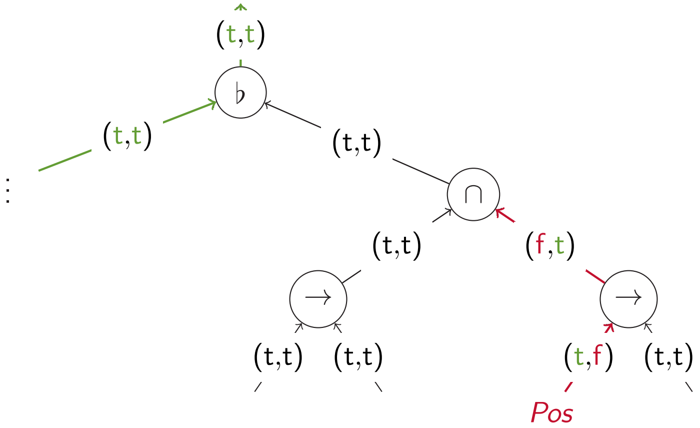
Constraint Graph



Reduction

- $\longrightarrow \flat_3 \blacktriangleleft (\mathit{false}) : \cdots,$
  $(((\lambda x.x + 1)\ @^{\flat_1}\ (\mathit{Even} \to \mathit{Even}))\ 0)\ @^{\flat_4}\ \mathit{Pos}$

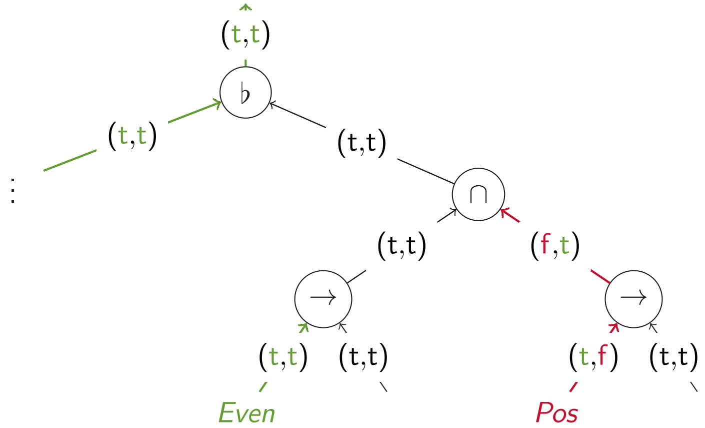
Constraint Graph



Reduction

- $\longrightarrow \flat_1 \blacktriangleleft (\flat_5 \to \flat_6) : \cdots,$
  $(((\lambda x.x + 1)\ (0\ @^{\flat_5}\ \mathit{Even}))\ @^{\flat_6}\ \mathit{Even})\ @^{\flat_4}\ \mathit{Pos}$

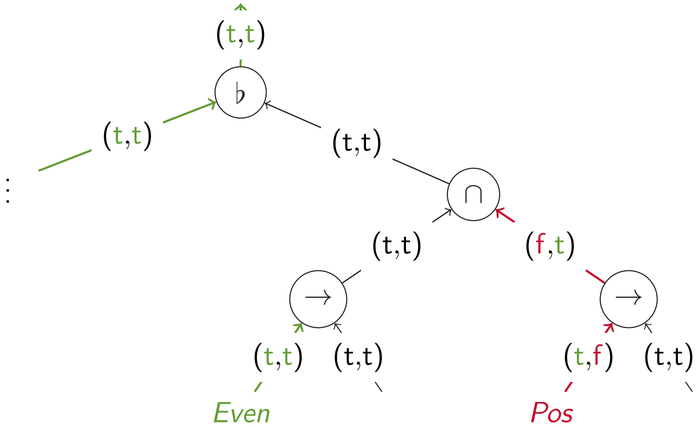
Constraint Graph



Reduction

- $\longrightarrow \flat_5 \blacktriangleleft (\mathit{true}) : \cdots ,$
  $(((\lambda x.x + 1)\ 0)\ @^{\flat_6}\ \mathit{Even})\ @^{\flat_4}\ \mathit{Pos}$

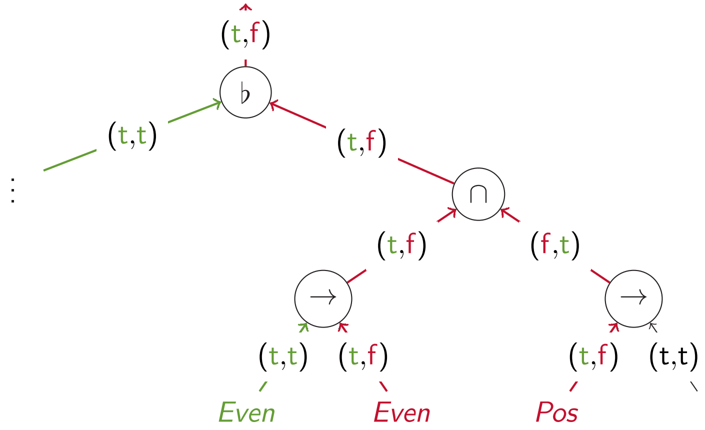
Constraint Graph



Reduction

- $\longrightarrow \cdots,$
  $(1 \ @^{\flat 6} \ \mathit{Even}) \ @^{\flat 4} \ \mathit{Pos}$

Constraint Graph



Reduction

- $\longrightarrow \flat_6 \blacktriangleleft (\mathit{false}) : \cdots,$
  $\mathit{blame}^{\flat}$

Intersection and Union Types

Intersection Type

- $\lambda x.x + 2 : \mathit{Even} \to \mathit{Even}$
- $\lambda x.x + 2 : \mathit{Odd} \to \mathit{Odd}$
- $\lambda x.x + 2 : \mathit{Even} \to \mathit{Even} \cap \mathit{Odd} \to \mathit{Odd}$

Union Type

- $\lambda x.x - 2 : \mathit{Even} \to \mathit{Even}$
- $\lambda x.x - 2 : \mathit{Even} \to \mathit{Even} \cup \mathit{Pos} \to \mathit{Pos}$

Flat Contract [Findler,Felleisen'02]

- $Pos = flat(\lambda x.x > 0)$
- $Even = flat(\lambda x.x\%2 = 0)$

Flat Contract [Findler,Felleisen'02]

- $Pos = flat(\lambda x.x > 0)$
- $Even = flat(\lambda x.x\%2 = 0)$

Assertion

- $1@Pos \longrightarrow 1$ ✓
- $0@Pos \longrightarrow$ ✗ blame subject 0

Flat Contract [Findler,Felleisen'02]

- $Pos = flat(\lambda x.x > 0)$
- $Even = flat(\lambda x.x\%2 = 0)$

Assertion

- $1@Pos \longrightarrow 1$ ✓
- $0@Pos \longrightarrow$ ✗ blame subject 0

Definition

- Subject V gets blamed for *Flat Contract* $flat(M)$ iff: $(M\ V) \longrightarrow^* false$

Higher-Order Contract [Findler,Felleisen'02]

- $Even \rightarrow Even$

Higher-Order Contract [Findler,Felleisen'02]

- $Even \rightarrow Even$

Assertion

- $((\lambda x.x + 1)@Even \rightarrow Even)\ 1 \longrightarrow^{*}$ ✗ blame context $\square$ 1

Definition

- Context gets <u>*blamed*</u> for $\mathcal{C} \rightarrow \mathcal{D}$ iff:
  Argument x gets *blamed* for $\mathcal{C}$ (as subject)
- Subject M gets <u>*blamed*</u> for $\mathcal{C} \rightarrow \mathcal{D}$ at $\square\ V$ iff:
  $\neg$ (Context gets *blamed* $\mathcal{C}$) $\wedge$ ($M\ V$ gets *blamed* $\mathcal{D}$)

Higher-Order Contract [Findler,Felleisen'02]

- $\mathit{Even} \rightarrow \mathit{Even}$

Assertion

- $((\lambda x.x + 1)@\mathit{Even} \rightarrow \mathit{Even})\ 1 \longrightarrow^*$ ✗ blame context $\square$ 1
- $((\lambda x.x + 1)@\mathit{Even} \rightarrow \mathit{Even})\ 2 \longrightarrow^*$ ✗ blame subject

Definition

- Context gets *blamed* for $\mathcal{C} \rightarrow \mathcal{D}$ iff:
  Argument x gets *blamed* for $\mathcal{C}$ (as subject)
- Subject M gets *blamed* for $\mathcal{C} \rightarrow \mathcal{D}$ at $\square$ V iff:
  $\neg$ (Context gets *blamed* $\mathcal{C}$) $\wedge$ (M V gets *blamed* $\mathcal{D}$)

Flat Contract

Examples

- $\mathit{Odd} \cup \mathit{Even}$

Flat Contract

Examples

- *Odd* $\cup$ *Even*

Flat Contract

- $\mathit{flat}(\lambda x.P) \cup \mathit{flat}(\lambda x.Q) \equiv \mathit{flat}(\lambda x.P \vee Q)$

Union Contract

Assertion

Let $mod3 = ((\lambda x.x\%3) \mathbin{@} (Even \rightarrow Even) \cup (Pos \rightarrow Pos))$

Definition

- Context gets *blamed* for $\mathcal{C} \cup \mathcal{D}$ iff:
  (Context gets *blamed* for $\mathcal{C}$) $\vee$ (Context gets *blamed* for $\mathcal{D}$)
- Subject M gets *blamed* for $\mathcal{C} \cup \mathcal{D}$ iff:
  (M gets *blamed* for $\mathcal{C}$) $\wedge$ (M gets *blamed* for $\mathcal{D}$)

Union Contract

Assertion

Let $mod3 = ((\lambda x.x\%3) \mathbin{@} (Even \rightarrow Even) \cup (Pos \rightarrow \underline{Pos}))$

- $(mod3\ 4) \longrightarrow^{*} 1$ ✓

Definition

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Union Contract

Assertion

Let $mod3 = ((\lambda x.x\%3) \mathbin{@} (Even \rightarrow Even) \cup (Pos \rightarrow \underline{Pos}))$

- $(mod3\ 4) \longrightarrow^* 1$ ✓
- $(mod3\ 1) \longrightarrow^*$ ✗ blame context $\Box$ 1

Definition

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  (M gets *blamed* for $\mathcal{C}$) $\wedge$ (M gets *blamed* for $\mathcal{D}$)

Union Contract

Assertion

Let $mod3 = ((\lambda x.x\%3) \; @ \; (Even \rightarrow Even) \cup (Pos \rightarrow \underline{Pos}))$

- $(mod3\ 4) \longrightarrow^{*} 1$ ✓
- $(mod3\ 1) \longrightarrow^{*}$ ✗ blame context $\square\ 1$
- $(mod3\ 6) \longrightarrow^{*}$ ✗ blame subject $(\lambda x.x\%3)$

Definition

- Context gets *blamed* for $\mathcal{C} \cup \mathcal{D}$ iff:
  (Context gets *blamed* for $\mathcal{C}$) $\vee$ (Context gets *blamed* for $\mathcal{D}$)
- Subject M gets *blamed* for $\mathcal{C} \cup \mathcal{D}$ iff:
  (M gets *blamed* for $\mathcal{C}$) $\wedge$ (M gets *blamed* for $\mathcal{D}$)

Contract Assertion

Evaluation Rule

$$\textsc{Assert}\quad \frac{\flat \notin \varsigma \qquad \varsigma' = \flat \blacktriangleleft (\flat) : \varsigma}{\varsigma, E[V @^{\flat} \mathcal{C}] \longrightarrow^{*} \varsigma', E[V @^{\flat} \mathcal{C}]}$$

Constraint Satisfaction

$$\textsc{C-Assert}\quad \frac{\mu(\flat.subject) \sqsupseteq \mu(\flat_1.subject) \qquad \mu(\flat.context) \sqsupseteq \mu(\flat_1.context)}{\mu \models \flat \blacktriangleleft (\flat_1)}$$

Constraint List

Constraint Satisfaction

CS-Empty

$$\mu \models \cdot$$

CS-Cons

$$\frac{\mu \models \kappa \qquad \mu \models \varsigma}{\mu \models \kappa : \varsigma}$$

Union Contract

Evaluation Rule

Union

$$\frac{\flat_1, \flat_2 \notin \varsigma \qquad \varsigma' = \flat \blacktriangleleft (\flat_1 \cup \flat_2) : \varsigma}{\varsigma, E[V \,@^{\flat}\, (\mathcal{C} \cup \mathcal{D})] \longrightarrow \varsigma', E[(V \,@^{\flat_1}\, \mathcal{C}) \,@^{\flat_2}\, \mathcal{D}]}$$

Constraint Satisfaction

C-Union

$$\frac{\begin{array}{c}\mu(\flat.\mathit{subject}) \sqsupseteq \mu(\flat_1.\mathit{subject} \vee \flat_2.\mathit{subject}) \\ \mu(\flat.\mathit{context}) \sqsupseteq \mu(\flat_1.\mathit{context} \wedge \flat_2.\mathit{context})\end{array}}{\mu \models \flat \blacktriangleleft \flat_1 \cup \flat_2}$$

Blame Calculation

Definition

ς is a *blame state* if there exists a top-level blame identifier such that

$$\mu(\flat.\mathit{subject}) \sqsupseteq \mathsf{f} \;\vee\; \mu(\flat.\mathit{context}) \sqsupseteq \mathsf{f}$$

$$\frac{\varsigma, M \longrightarrow^{*} \varsigma', N \qquad \varsigma \text{ is not a blame state}}{\varsigma, M \longmapsto \varsigma', N}$$

$$\frac{\varsigma \text{ is blame state for } \flat}{\varsigma, M \longmapsto \varsigma, \mathit{blame}^{\flat}}$$

Example Reduction

Reduction

- $\cdot,$
  $((\lambda x.x + 1) \;@^{\flat}\; ((\mathit{Even} \to \mathit{Even}) \cap (\mathit{Pos} \to \mathit{Pos}))) \; 0$

Example Reduction

Reduction

- $\cdot,$
  $((\lambda x.x + 1) \; @^{\flat} \; ((\mathit{Even} \to \mathit{Even}) \cap (\mathit{Pos} \to \mathit{Pos}))) \; 0$
- $\longrightarrow \flat\blacktriangleleft(\flat_0) : \cdot,$
  $((\lambda x.x + 1) \; @^{\flat_0} \; ((\mathit{Even} \to \mathit{Even}) \cap (\mathit{Pos} \to \mathit{Pos}))) \; 0$

Example Reduction

Reduction

- $\cdot,$
  $((\lambda x.x+1) \text{ @}^{\flat} ((\mathit{Even} \to \mathit{Even}) \cap (\mathit{Pos} \to \mathit{Pos}))) \; 0$
- $\longrightarrow \flat \blacktriangleleft (\flat_0) : \cdot,$
  $((\lambda x.x+1) \text{ @}^{\flat_0} ((\mathit{Even} \to \mathit{Even}) \cap (\mathit{Pos} \to \mathit{Pos}))) \; 0$
- $\longrightarrow \flat_0 \blacktriangleleft (\flat_1 \cap \flat_2) : \cdots,$
  $(((\lambda x.x+1) \text{ @}^{\flat_1} (\mathit{Even} \to \mathit{Even})) \text{ @}^{\flat_2} (\mathit{Pos} \to \mathit{Pos})) \; 0$

Example Reduction

Reduction

- $\cdot,$
  $((\lambda x.x+1) \mathbin{@^\flat} ((\mathit{Even} \to \mathit{Even}) \cap (\mathit{Pos} \to \mathit{Pos})))\ 0$
- $\longrightarrow \flat \blacktriangleleft (\flat_0) : \cdot,$
  $((\lambda x.x+1) \mathbin{@^{\flat_0}} ((\mathit{Even} \to \mathit{Even}) \cap (\mathit{Pos} \to \mathit{Pos})))\ 0$
- $\longrightarrow \flat_0 \blacktriangleleft (\flat_1 \cap \flat_2) : \cdots,$
  $(((\lambda x.x+1) \mathbin{@^{\flat_1}} (\mathit{Even} \to \mathit{Even})) \mathbin{@^{\flat_2}} (\mathit{Pos} \to \mathit{Pos}))\ 0$
- $\longrightarrow \flat_2 \blacktriangleleft (\flat_3 \to \flat_4) : \cdots,$
  $(((\lambda x.x+1) \mathbin{@^{\flat_1}} (\mathit{Even} \to \mathit{Even}))\ (0 \mathbin{@^{\flat_3}} \mathit{Pos})) \mathbin{@^{\flat_4}} \mathit{Pos}$

Example Reduction

Reduction

- $\cdot,$
  $((\lambda x.x + 1) \mathbin{@^{\flat}} ((\mathit{Even} \to \mathit{Even}) \cap (\mathit{Pos} \to \mathit{Pos}))) \; 0$
- $\longrightarrow \flat \blacktriangleleft (\flat_0) : \cdot,$
  $((\lambda x.x + 1) \mathbin{@^{\flat_0}} ((\mathit{Even} \to \mathit{Even}) \cap (\mathit{Pos} \to \mathit{Pos}))) \; 0$
- $\longrightarrow \flat_0 \blacktriangleleft (\flat_1 \cap \flat_2) : \cdots,$
  $(((\lambda x.x + 1) \mathbin{@^{\flat_1}} (\mathit{Even} \to \mathit{Even})) \mathbin{@^{\flat_2}} (\mathit{Pos} \to \mathit{Pos})) \; 0$
- $\longrightarrow \flat_2 \blacktriangleleft (\flat_3 \to \flat_4) : \cdots,$
  $(((\lambda x.x + 1) \mathbin{@^{\flat_1}} (\mathit{Even} \to \mathit{Even})) \; (0 \mathbin{@^{\flat_3}} \mathit{Pos})) \mathbin{@^{\flat_4}} \mathit{Pos}$
- $\longrightarrow \flat_3 \blacktriangleleft (\mathit{false}) : \cdots,$
  $(((\lambda x.x + 1) \mathbin{@^{\flat_1}} (\mathit{Even} \to \mathit{Even})) \; 0) \mathbin{@^{\flat_4}} \mathit{Pos}$

Example Reduction

Reduction

- $\longrightarrow \flat \blacktriangleleft (\flat_0) : \cdot,$
  $((\lambda x.x + 1) \ @^{\flat_0} \ ((Even \to Even) \cap (Pos \to Pos))) \ 0$
- $\longrightarrow \flat_0 \blacktriangleleft (\flat_1 \cap \flat_2) : \cdots,$
  $(((\lambda x.x + 1) \ @^{\flat_1} \ (Even \to Even)) \ @^{\flat_2} \ (Pos \to Pos)) \ 0$
- $\longrightarrow \flat_2 \blacktriangleleft (\flat_3 \to \flat_4) : \cdots,$
  $(((\lambda x.x + 1) \ @^{\flat_1} \ (Even \to Even)) \ (0 \ @^{\flat_3} \ Pos)) \ @^{\flat_4} \ Pos$
- $\longrightarrow \flat_3 \blacktriangleleft (\mathit{false}) : \cdots,$
  $(((\lambda x.x + 1) \ @^{\flat_1} \ (Even \to Even)) \ 0) \ @^{\flat_4} \ Pos$
- $\longrightarrow \flat_1 \blacktriangleleft (\flat_5 \to \flat_6) : \cdots,$
  $(((\lambda x.x + 1) \ (0 \ @^{\flat_5} \ Even)) \ @^{\flat_6} \ Even) \ @^{\flat_4} \ Pos$

Example Reduction

Reduction

- $\longrightarrow \flat_0 \blacktriangleleft (\flat_1 \cap \flat_2) : \cdots,$
  $(((\lambda x.x + 1) @^{\flat_1} (\mathit{Even} \to \mathit{Even})) @^{\flat_2} (\mathit{Pos} \to \mathit{Pos})) \; 0$
- $\longrightarrow \flat_2 \blacktriangleleft (\flat_3 \to \flat_4) : \cdots,$
  $(((\lambda x.x + 1) @^{\flat_1} (\mathit{Even} \to \mathit{Even})) \; (0 @^{\flat_3} \mathit{Pos})) @^{\flat_4} \mathit{Pos}$
- $\longrightarrow \flat_3 \blacktriangleleft (\mathit{false}) : \cdots,$
  $(((\lambda x.x + 1) @^{\flat_1} (\mathit{Even} \to \mathit{Even})) \; 0) @^{\flat_4} \mathit{Pos}$
- $\longrightarrow \flat_1 \blacktriangleleft (\flat_5 \to \flat_6) : \cdots,$
  $(((\lambda x.x + 1) \; (0 @^{\flat_5} \mathit{Even})) @^{\flat_6} \mathit{Even}) @^{\flat_4} \mathit{Pos}$
- $\longrightarrow \flat_5 \blacktriangleleft (\mathit{true}) : \cdots,$
  $(((\lambda x.x + 1) \; 0) @^{\flat_6} \mathit{Even}) @^{\flat_4} \mathit{Pos}$

Example Reduction

Reduction

- $\longrightarrow \flat_2 \blacktriangleleft (\flat_3 \to \flat_4) : \cdots,$
  $(((\lambda x.x + 1) \ @^{\flat_1} \ (Even \to Even)) \ (0 \ @^{\flat_3} \ Pos)) \ @^{\flat_4} \ Pos$
- $\longrightarrow \flat_3 \blacktriangleleft (false) : \cdots,$
  $(((\lambda x.x + 1) \ @^{\flat_1} \ (Even \to Even)) \ 0) \ @^{\flat_4} \ Pos$
- $\longrightarrow \flat_1 \blacktriangleleft (\flat_5 \to \flat_6) : \cdots,$
  $(((\lambda x.x + 1) \ (0 \ @^{\flat_5} \ Even)) \ @^{\flat_6} \ Even) \ @^{\flat_4} \ Pos$
- $\longrightarrow \flat_5 \blacktriangleleft (true) : \cdots,$
  $(((\lambda x.x + 1) \ 0) \ @^{\flat_6} \ Even) \ @^{\flat_4} \ Pos$
- $\longrightarrow \cdots,$
  $(1 \ @^{\flat_6} \ Even) \ @^{\flat_4} \ Pos$

Example Reduction

Reduction

- $\longrightarrow \flat_3 \blacktriangleleft (\mathit{false}) : \cdots,$
  $(((\lambda x.x + 1) \mathbin{@^{\flat_1}} (\mathit{Even} \rightarrow \mathit{Even}))\ 0) \mathbin{@^{\flat_4}} \mathit{Pos}$
- $\longrightarrow \flat_1 \blacktriangleleft (\flat_5 \rightarrow \flat_6) : \cdots,$
  $(((\lambda x.x + 1)\ (0 \mathbin{@^{\flat_5}} \mathit{Even})) \mathbin{@^{\flat_6}} \mathit{Even}) \mathbin{@^{\flat_4}} \mathit{Pos}$
- $\longrightarrow \flat_5 \blacktriangleleft (\mathit{true}) : \cdots,$
  $(((\lambda x.x + 1)\ 0) \mathbin{@^{\flat_6}} \mathit{Even}) \mathbin{@^{\flat_4}} \mathit{Pos}$
- $\longrightarrow \cdots,$
  $(1 \mathbin{@^{\flat_6}} \mathit{Even}) \mathbin{@^{\flat_4}} \mathit{Pos}$
- $\longrightarrow \flat_6 \blacktriangleleft (\mathit{false}) : \cdots,$
  $\mathit{blame}^{\flat}$

Example Reduction

Reduction

- $\longrightarrow \flat_1 \blacktriangleleft (\flat_5 \rightarrow \flat_6) : \cdots,$
  $(((\lambda x.x + 1)\ (0\ @^{\flat_5}\ \mathit{Even}))\ @^{\flat_6}\ \mathit{Even})\ @^{\flat_4}\ \mathit{Pos}$
- $\longrightarrow \flat_5 \blacktriangleleft (\mathit{true}) : \cdots,$
  $(((\lambda x.x + 1)\ 0)\ @^{\flat_6}\ \mathit{Even})\ @^{\flat_4}\ \mathit{Pos}$
- $\longrightarrow \cdots,$
  $(1\ @^{\flat_6}\ \mathit{Even})\ @^{\flat_4}\ \mathit{Pos}$
- $\longrightarrow \flat_6 \blacktriangleleft (\mathit{false}) : \cdots,$
  $\mathit{blame}^{\flat}$

Example Reduction

Reduction

- $\longrightarrow \flat_5 \blacktriangleleft (\mathit{true}) : \cdots,$
  $(((\lambda x.x + 1)\ 0)\ @^{\flat_6}\ \mathit{Even})\ @^{\flat_4}\ \mathit{Pos}$
- $\longrightarrow \cdots,$
  $(1\ @^{\flat_6}\ \mathit{Even})\ @^{\flat_4}\ \mathit{Pos}$
- $\longrightarrow \flat_6 \blacktriangleleft (\mathit{false}) : \cdots,$
  $\mathit{blame}^{\flat}$

Example Reduction

Reduction

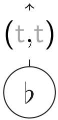
- $\longrightarrow \cdots,$
  $(1 \mathbin{@^{\flat_6}} \mathit{Even}) \mathbin{@^{\flat_4}} \mathit{Pos}$
- $\longrightarrow \flat_6 \blacktriangleleft (\mathit{false}) : \cdots,$
  $\mathit{blame}^{\flat}$

Example Reduction

Reduction

- $\longrightarrow \flat_6 \blacktriangleleft(\mathit{false}) : \cdots,$
  $\mathit{blame}^{\flat}$

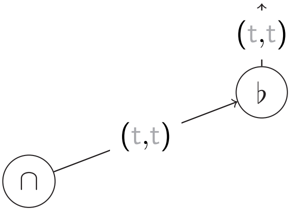
Constraint Graph



Example

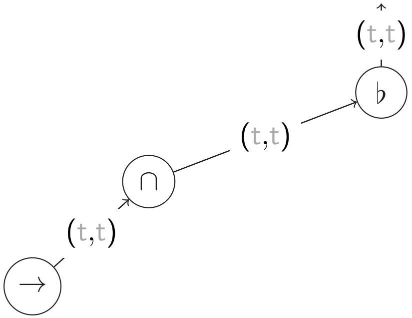
- $((\lambda x.x+1)\ @^{\flat}\ ((\mathit{Even} \rightarrow \mathit{Even}) \cap (\mathit{Pos} \rightarrow \mathit{Pos})))\ 1 \longrightarrow^{*} 2$ ✓

Constraint Graph



Example

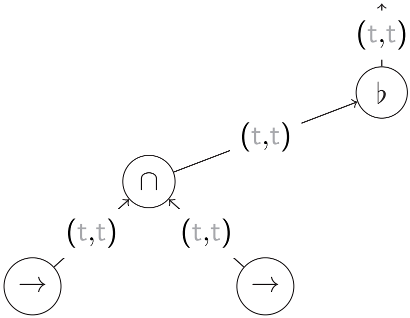
- $((\lambda x.x + 1)\ @^{\flat}\ ((Even \rightarrow Even) \cap (Pos \rightarrow Pos)))\ 1 \longrightarrow^{*} 2$ ✓



Example

- $((\lambda x.x+1)\ @^{\flat}\ ((Even \to Even) \cap (Pos \to Pos)))\ 1 \longrightarrow^{*} 2$ ✓

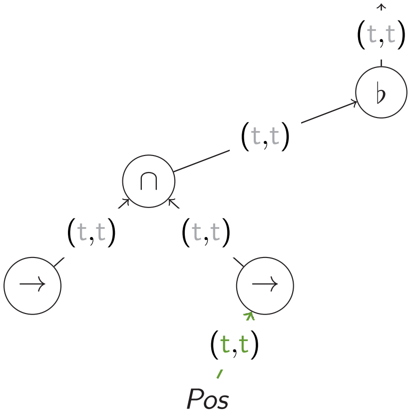
Constraint Graph



Example

- $((\lambda x.x + 1)\ @^{\flat}\ ((\mathit{Even} \to \mathit{Even}) \cap (\mathit{Pos} \to \mathit{Pos})))\ 1 \longrightarrow^{*} 2$ ✓

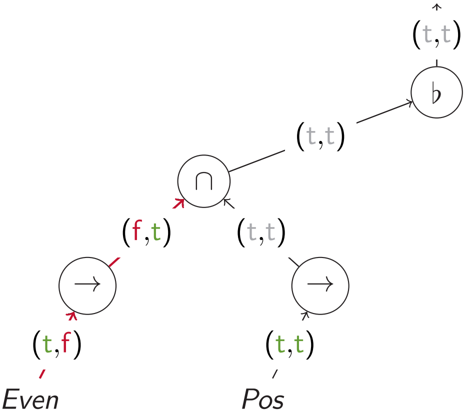
Constraint Graph



Example

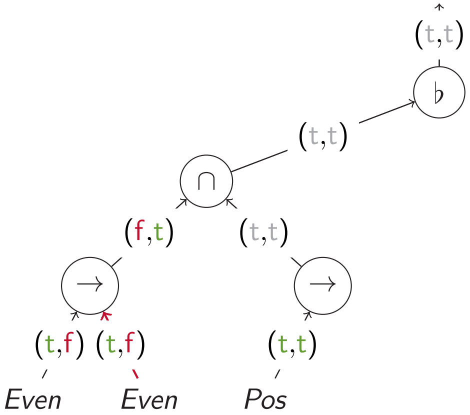
- $((\lambda x.x+1)\ @^{\flat}\ ((Even \to Even) \cap (Pos \to Pos)))\ 1 \longrightarrow^{*} 2$ ✓

Constraint Graph



Example

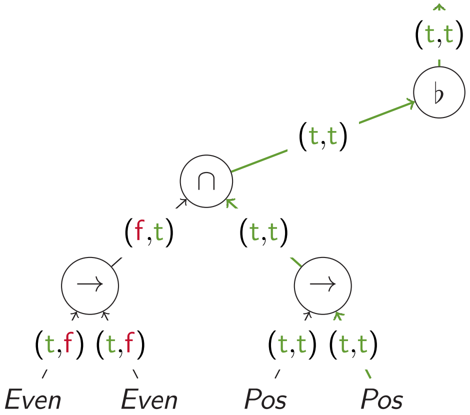
- $((\lambda x.x + 1)\ @^{\flat}\ ((\mathit{Even} \to \mathit{Even}) \cap (\mathit{Pos} \to \mathit{Pos})))\ 1 \longrightarrow^{*} 2$ ✓



Example

- $((\lambda x.x+1) \; @^\flat \; ((Even \to Even) \cap (Pos \to Pos))) \; 1 \longrightarrow^* 2$ ✓

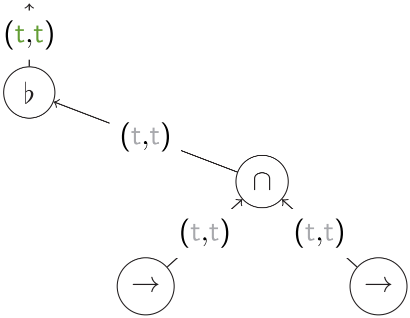
Constraint Graph



Example

- $((\lambda x.x + 1) \; @^{\flat} \; ((\mathit{Even} \rightarrow \mathit{Even}) \cap (\mathit{Pos} \rightarrow \mathit{Pos}))) \; 1 \longrightarrow^{*} 2$ ✓

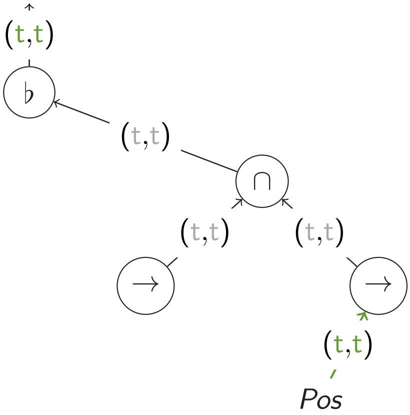
Constraint Graph



Example

- $((\lambda x.x + 1)\ @^\flat\ ((Even \to Even) \cap (Pos \to Pos)))\ 1 \longrightarrow^* 2$ ✓
- $((\lambda x.x + 1)\ @^\flat\ ((Even \to Even) \cap (Pos \to Pos)))\ 2 \longrightarrow^*$ ✗

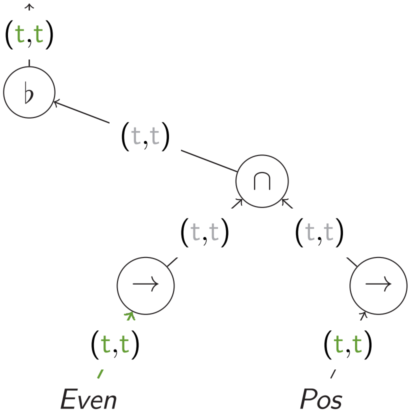
Constraint Graph



Example

- $((\lambda x.x+1)\ @^{\flat}\ ((Even \to Even) \cap (Pos \to Pos)))\ 1 \longrightarrow^{*} 2$ ✓
- $((\lambda x.x+1)\ @^{\flat}\ ((Even \to Even) \cap (Pos \to Pos)))\ 2 \longrightarrow^{*}$ ✗

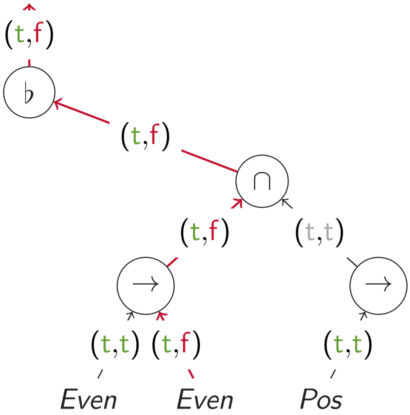
Constraint Graph



Example

- $((\lambda x.x + 1) \ @^\flat \ ((\mathit{Even} \to \mathit{Even}) \cap (\mathit{Pos} \to \mathit{Pos}))) \ 1 \longrightarrow^* 2$ ✓
- $((\lambda x.x + 1) \ @^\flat \ ((\mathit{Even} \to \mathit{Even}) \cap (\mathit{Pos} \to \mathit{Pos}))) \ 2 \longrightarrow^*$ ✗

Constraint Graph



Example

- $((\lambda x.x + 1) \text{ @}^\flat ((Even \to Even) \cap (Pos \to Pos))) \ 1 \longrightarrow^* 2$ ✓
- $((\lambda x.x + 1) \text{ @}^\flat ((Even \to Even) \cap (Pos \to Pos))) \ 2 \longrightarrow^*$ ✗

Technical Results

Definition (Contract Satisfaction)

The semantics of a contract $\mathcal{C}$ defines

1. a set $[\![\mathcal{C}]\!]^+$ of *closed terms* (subjects) that *satisfy* $\mathcal{C}$
2. a set $[\![\mathcal{C}]\!]^-$ of *closed contexts* that *respect* $\mathcal{C}$

The definition is mutually inductive on the structure of $\mathcal{C}$.

Technical Results (cont'd)

Theorem (Contract soundness for expressions)

For all $M, \mathcal{C}, \flat.\ M \mathbin{@^\flat} \mathcal{C} \in [\![\mathcal{C}]\!]^+$

Theorem (Contract soundness for contexts)

For all $\mathcal{L}, \mathcal{C}, \flat.\ \mathcal{L}[\square \mathbin{@^\flat} \mathcal{C}] \in [\![\mathcal{C}]\!]^-$

Technical Results (cont'd)

Theorem (Subject blame soundness)

Suppose that $M \in \llbracket \mathcal{C} \rrbracket^{+}$.
If $\varsigma, E[M\,@^{\flat}\,\mathcal{C}] \longmapsto^{*} \varsigma', N$ *then* $\llbracket \varsigma' \rrbracket(\flat, \mathit{subject}) \sqsubseteq \mathrm{t}$.

Theorem (Context blame soundness)

Suppose that $\mathcal{L} \in \llbracket \mathcal{C} \rrbracket^{-}$.
If $\varsigma, \mathcal{L}[M\,@^{\flat}\,\mathcal{C}] \longmapsto^{*} \varsigma', N,$ *then* $\llbracket \varsigma' \rrbracket(\flat, \mathit{context}) \sqsubseteq \mathrm{t}$.