Blame Assignment for Higher-Order Contracts with Intersection and Union

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Higher-Order Contracts

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- $Even = flat(\lambda x.x\%2 = 0)$
- $Odd = flat(\lambda x.x\%2 = 1)$

Assertion $(Even \rightarrow Even)$

Let $add2Even = ((\lambda x.x + 2)$ @ $(Even \rightarrow Even))$ Let $add2Even = ((\lambda x.x + 2)$ @ $(Even \rightarrow Even))$ Let $add2Even = ((\lambda x.x + 2)$ @ $(Even \rightarrow Even))$

- $(add2Even\ 2) \longrightarrow^* 4$ ✓
- $(add2Even\ 1) \longrightarrow^*$ ✗ blame context ☐ 1

Assertion $(Odd \rightarrow Odd)$

Let $add2Odd = ((\lambda x.x + 2)$ @ $(Odd \rightarrow Odd))$ Let $add2Odd = ((\lambda x.x + 2)$ @ $(Odd \rightarrow Odd))$ Let $add2Odd = ((\lambda x.x + 2)$ @ $(Odd \rightarrow Odd))$

- $(add2Odd\ 1) \longrightarrow^* 3$ ✓
- $(add2Odd\ 2) \longrightarrow^*$ ✗ blame context ☐ 2

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Combination of Contracts

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Observation

- $\lambda x.x + 2$ works for *even* and *odd* arguments
- $\lambda x.x + 2$ fulfills $Odd \rightarrow Odd$ **and** $Even \rightarrow Even$
- How can we express that with a single contract?

Intersection Contract!

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Inspiration

Intersection Type

- $V : S \cap T$
- Models overloading
- Models multiple inheritances

Union Type

- $V : S \cup T$
- Dual of intersection type
- Domain of overloaded functions

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This Work

- Extend higher-order contracts with intersection and union
- Specification based on the type theoretic construction

Assertion $(Even \to Even) \cap (Odd \to Odd)$

Let $add2 = ((\lambda x.x + 2) \mathbin{@} (Even \to Even) \cap (Odd \to Odd))$

- $(add2\ 2) \longrightarrow^* 4$ ✓
- $(add2\ 1) \longrightarrow^* 3$ ✓

No blame because of the intersection contract!

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Flat Contract

- $Even = flat(\lambda x.x\%2 = 0)$
- $Odd = flat(\lambda x.x\%2 = 1)$
- $Pos = flat(\lambda x.x > 0)$

Examples

- $Pos \cap Even$

Flat Contract

- $flat(\lambda x.P) \cap flat(\lambda x.Q) \equiv flat(\lambda x.P \wedge Q)$

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Intersection Contract

Assertion

Let $add1 = ((\lambda x.x + 1)$ @ $(Even \rightarrow Even) \cap (Pos \rightarrow Pos))$ Let $add1 = ((\lambda x.x + 1)$ @ $(Even \rightarrow Even) \cap (Pos \rightarrow Pos))$ Let $add1 = ((\lambda x.x + 1)$ @ $(Even \rightarrow Even) \cap (Pos \rightarrow Pos))$ Let $add1 = ((\lambda x.x + 1)$ @ $(Even \rightarrow Even) \cap (Pos \rightarrow Pos))$

- $(add1\ 3) \longrightarrow^* 4$ ✓
- $(add1\ {-1}) \longrightarrow^*$ ✗ blame context $\square\ {-1}$
- $(add1\ 2) \longrightarrow^*$ ✗ blame subject $(\lambda x.x + 1)$

Definition

- Context gets *blamed* for $\mathcal{C} \cap \mathcal{D}$ iff:
  (Context gets *blamed* for $\mathcal{C}$) $\wedge$ (Context gets *blamed* for $\mathcal{D}$)
- Subject M gets *blamed* for $\mathcal{C} \cap \mathcal{D}$ iff:
  (M gets *blamed* for $\mathcal{C}$) $\vee$ (M gets *blamed* for $\mathcal{D}$)

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Contract Assertion

Example

- $((\lambda x.x + 1)$ @ $(EvenEven \rightarrow Even) \cap (PosPos \rightarrow PosPos))\ 3 \longrightarrow^* 4$ ✓

- A failing contract must not signal a violation immediately
- Violation depends on combinations of failures in different sub-contracts
- Contract assertion must connect each contract with the enclosing operations

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Operational Semantics

Reduction Relation

$$\varsigma, M \longrightarrow \varsigma', N$$

- M, N expressions
- ς list of constraints
- One constraint for each contract operator $\rightarrow$, $\cap$, $\cup$
- One constraint for each flat contract
- Blame calculation from a list of constraints

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Flat Contract

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Evaluation Rule

$$\frac{\textsc{Flat}\quad M\ V \longrightarrow^* W \qquad \varsigma' = \flat \blacktriangleleft (W) : \varsigma}{\varsigma, E[V \mathbin{@^\flat} \mathit{flat}(M)] \longrightarrow \varsigma', E[V]}$$

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Interpretation

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Interpretation of a constraint list

$$\mu \in (\llparenthesis \flat \rrparenthesis \times \{\mathit{subject}, \mathit{context}\}) \to \mathbb{B}$$

- An interpretation μ is a mapping from blame label $\flat$ to records of elements of $\mathbb{B} = \{\mathsf{t}, \mathsf{f}\}$, order $\mathsf{t} \sqsubset \mathsf{f}$
- Ordering reflects gathering of information with each execution step
- Each blame label $\flat$ is associated with two truth values, $\flat.\mathit{subject}$ and $\flat.\mathit{context}$

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Flat Contract (cont'd)

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Evaluation Rule

$$\frac{\textsc{Flat}\quad M\ V \longrightarrow^* W \qquad \varsigma' = \flat \blacktriangleleft (W) : \varsigma}{\varsigma, E[V \mathbin{@^\flat} \mathit{flat}(M)] \longrightarrow \varsigma', E[V]}$$

Constraint Satisfaction

$$\frac{\textsc{C-Flat}\quad \mu(\flat.\mathit{subject}) \sqsupseteq W \qquad \mu(\flat.\mathit{context}) \sqsupseteq \mathsf{t}}{\mu \models \flat \blacktriangleleft W}$$

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Blame Calculation

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Definition

ς is a *blame state* if there exists a top-level blame label such that

$$\mu(\flat.\mathit{subject}) \sqsupseteq \mathsf{f} \;\vee\; \mu(\flat.\mathit{context}) \sqsupseteq \mathsf{f}$$

- Evaluation stops if a blame state is reached.

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Function Contract

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Evaluation Rule

Function

$$\frac{\flat_1, \flat_2 \notin \varsigma \qquad \varsigma' = \flat \blacktriangleleft (\flat_1 \rightarrow \flat_2) : \varsigma}{\varsigma, E[(V \mathbin{@^{\flat}} (\mathcal{C} \rightarrow \mathcal{D}))\; W] \longrightarrow \varsigma', E[(V\; (W \mathbin{@^{\flat_1}} \mathcal{C})) \mathbin{@^{\flat_2}} \mathcal{D}]}$$

Constraint Satisfaction

C-Function

$$\frac{\begin{array}{c}\mu(\flat.\mathit{subject}) \sqsupseteq \mu(\flat_1.\mathit{context} \wedge (\flat_1.\mathit{subject} \Rightarrow \flat_2.\mathit{subject})) \\ \mu(\flat.\mathit{context}) \sqsupseteq \mu(\flat_1.\mathit{subject} \wedge \flat_2.\mathit{context})\end{array}}{\mu \models \flat \blacktriangleleft \flat_1 \rightarrow \flat_2}$$

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Intersection Contract

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Evaluation Rule

Intersection

$$\frac{\flat_1, \flat_2 \notin \varsigma \qquad \varsigma' = \flat \blacktriangleleft (\flat_1 \cap \flat_2) : \varsigma}{\varsigma, E[(V \mathbin{@^{\flat}} (Q \cap R))\; W] \longrightarrow \varsigma', E[((V \mathbin{@^{\flat_1}} Q) \mathbin{@^{\flat_2}} R)\; W]}$$

Constraint Satisfaction

C-Intersection

$$\frac{\begin{array}{c}\mu(\flat.\mathit{subject}) \sqsupseteq \mu(\flat_1.\mathit{subject} \wedge \flat_2.\mathit{subject}) \\ \mu(\flat.\mathit{context}) \sqsupseteq \mu(\flat_1.\mathit{context} \vee \flat_2.\mathit{context})\end{array}}{\mu \models \flat \blacktriangleleft \flat_1 \cap \flat_2}$$

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Union Contract

- Dual of intersection contract
- Exchange $\wedge$ and $\vee$ in the blame calculation
- *Delayed* evaluation changes to an *immediate* evaluation

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In the Paper

Technical Results

- Contract Rewriting
- Deterministic and nondeterministic specification of contract monitoring
- Denotational specification of the semantics of contracts
- Theorems for contract and blame soundness

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Conclusion

- Intersection and union contracts provide dynamic guarantees equivalent to their type-theoretic counterparts
- Constraint-based blame calculation enables higher-order contracts with unrestricted intersection and union
- Formal basis of *TreatJS*, a language embedded, higher-order contract system implemented for JavaScript

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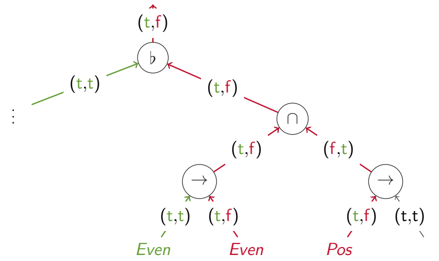
Constraint Graph



Reduction

- ς,
  $((\lambda x.x+1)\ @^{\flat}\ ((Even \rightarrow Even) \cap (Pos \rightarrow Pos)))\ 0$
- $\longrightarrow \flat \blacktriangleleft (\flat_1 \cap \flat_2) : \cdots,$
  $(((\lambda x.x+1)\ @^{\flat_1}\ (Even \rightarrow Even))\ @^{\flat_2}\ (Pos \rightarrow Pos))\ 0$

- $\longrightarrow \flat_2 \blacktriangleleft (\flat_3 \rightarrow \flat_4) : \cdots,$
  $(((\lambda x.x+1)\ @^{\flat_1}\ (Even \rightarrow Even))\ (0\ @^{\flat_3}\ Pos))\ @^{\flat_4}\ Pos$
- $\longrightarrow \flat_3 \blacktriangleleft (false) : \cdots,$
  $(((\lambda x.x+1)\ @^{\flat_1}\ (Even \rightarrow Even))\ 0)\ @^{\flat_4}\ Pos$
- $\longrightarrow \flat_1 \blacktriangleleft (\flat_5 \rightarrow \flat_6) : \cdots,$
  $(((\lambda x.x+1)\ (0\ @^{\flat_5}\ Even))\ @^{\flat_6}\ Even)\ @^{\flat_4}\ Pos$
- $\longrightarrow \flat_5 \blacktriangleleft (true) : \cdots,$
  $(((\lambda x.x+1)\ 0)\ @^{\flat_6}\ Even)\ @^{\flat_4}\ Pos$

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Intersection and Union Types

Intersection Type

- $\lambda x.x+2 : Even \rightarrow Even$
- $\lambda x.x+2 : Odd \rightarrow Odd$
- $\lambda x.x+2 : Even \rightarrow Even \cap Odd \rightarrow Odd$

Union Type

- $\lambda x.x-2 : Even \rightarrow Even$
- $\lambda x.x-2 : Even \rightarrow Even \cup Pos \rightarrow Pos$

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Flat Contract [Findler,Felleisen'02]

- $Pos = flat(\lambda x.x > 0)$
- $Even = flat(\lambda x.x\%2 = 0)$

Assertion

- $1@Pos \longrightarrow 1$ ✓
- $0@Pos \longrightarrow$ ✗ blame subject 0

Definition

- Subject V gets blamed for *Flat Contract* $flat(M)$ iff:
  $(M\ V) \longrightarrow^* false$

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Higher-Order Contract [Findler,Felleisen'02]

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- $Even \rightarrow Even$

Assertion

- $((\lambda x.x+1)@Even \rightarrow Even)\ 1 \longrightarrow^{*}$ ✗ blame context $\square\ 1$
- $((\lambda x.x+1)@Even \rightarrow Even)\ 2 \longrightarrow^{*}$ ✗ blame subject

Definition

- Context gets *blamed* for $\mathcal{C} \rightarrow \mathcal{D}$ iff:
  Argument x gets *blamed* for $\mathcal{C}$ (as subject)
- Subject M gets *blamed* for $\mathcal{C} \rightarrow \mathcal{D}$ at $\square\ V$ iff:
  $\neg$ (Context gets *blamed* $\mathcal{C}$) $\wedge$ ($M\ V$ gets *blamed* $\mathcal{D}$)

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Flat Contract

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Examples

- $Odd \cup Even$

Flat Contract

- $flat(\lambda x.P) \cup flat(\lambda x.Q) \equiv flat(\lambda x.P \vee Q)$

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Union Contract

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Assertion

Let $mod3 = ((\lambda x.x\%3)\ @\ (Even \rightarrow Even) \cup (Pos \rightarrow Pos))$ Let $mod3 = ((\lambda x.x\%3)\ @\ (Even \rightarrow Even) \cup (Pos \rightarrow \underline{Pos}))$ Let $mod3 = ((\lambda x.x\%3)\ @\ (Even \rightarrow Even) \cup (Pos \rightarrow \underline{Pos}))$ Let $mod3 = ((\lambda x.x\%3)\ @\ (Even \rightarrow Even) \cup (Pos \rightarrow \underline{Pos}))$

- $(mod3\ 4) \longrightarrow^{*} 1$ ✓
- $(mod3\ 1) \longrightarrow^{*}$ ✗ blame context $\square\ 1$
- $(mod3\ 6) \longrightarrow^{*}$ ✗ blame subject $(\lambda x.x\%3)$

Definition

- Context gets *blamed* for $\mathcal{C} \cup \mathcal{D}$ iff:
  (Context gets *blamed* for $\mathcal{C}$) $\vee$ (Context gets *blamed* for $\mathcal{D}$)
- Subject M gets *blamed* for $\mathcal{C} \cup \mathcal{D}$ iff:
  (M gets *blamed* for $\mathcal{C}$) $\wedge$ (M gets *blamed* for $\mathcal{D}$)

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Contract Assertion

Evaluation Rule

$$\textsc{Assert}\quad \frac{\flat \notin \varsigma \qquad \varsigma' = \flat\blacktriangleleft(\flat) : \varsigma}{\varsigma, E[V \mathbin{@^{\flat}} \mathcal{C}] \longrightarrow^{*} \varsigma', E[V \mathbin{@^{\flat}} \mathcal{C}]}$$

Constraint Satisfaction

$$\textsc{C-Assert}\quad \frac{\mu(\flat.\mathit{subject}) \sqsupseteq \mu(\flat_1.\mathit{subject}) \qquad \mu(\flat.\mathit{context}) \sqsupseteq \mu(\flat_1.\mathit{context})}{\mu \models \flat\blacktriangleleft(\flat_1)}$$

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Constraint List

Constraint Satisfaction

$$\textsc{CS-Empty}\quad \mu \models \cdot \qquad\qquad \textsc{CS-Cons}\quad \frac{\mu \models \kappa \qquad \mu \models \varsigma}{\mu \models \kappa : \varsigma}$$

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Union Contract

Evaluation Rule

$$\textsc{Union}\quad \frac{\flat_1, \flat_2 \notin \varsigma \qquad \varsigma' = \flat\blacktriangleleft(\flat_1 \cup \flat_2) : \varsigma}{\varsigma, E[V \mathbin{@^{\flat}} (\mathcal{C} \cup \mathcal{D})] \longrightarrow \varsigma', E[(V \mathbin{@^{\flat_1}} \mathcal{C}) \mathbin{@^{\flat_2}} \mathcal{D}]}$$

Constraint Satisfaction

$$\textsc{C-Union}\quad \frac{\begin{array}{c}\mu(\flat.\mathit{subject}) \sqsupseteq \mu(\flat_1.\mathit{subject} \vee \flat_2.\mathit{subject}) \\ \mu(\flat.\mathit{context}) \sqsupseteq \mu(\flat_1.\mathit{context} \wedge \flat_2.\mathit{context})\end{array}}{\mu \models \flat \blacktriangleleft \flat_1 \cup \flat_2}$$

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Blame Calculation

Definition

ς is a *blame state* if there exists a top-level blame identifier such that

$$\mu(\flat.\mathit{subject}) \sqsupseteq \mathsf{f} \ \vee\ \mu(\flat.\mathit{context}) \sqsupseteq \mathsf{f}$$

$$\frac{\varsigma, M \longrightarrow^{*} \varsigma', N \qquad \varsigma \text{ is not a blame state}}{\varsigma, M \longmapsto \varsigma', N} \qquad \frac{\varsigma \text{ is blame state for } \flat}{\varsigma, M \longmapsto \varsigma, \mathit{blame}^{\flat}}$$

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Example Reduction

Reduction

- $\cdot,$
  $((\lambda x.x + 1)\ @^{\flat}\ ((\mathit{Even} \to \mathit{Even}) \cap (\mathit{Pos} \to \mathit{Pos})))\ 0$
- $\longrightarrow \flat \blacktriangleleft (\flat_0) : \cdot,$
  $((\lambda x.x + 1)\ @^{\flat_0}\ ((\mathit{Even} \to \mathit{Even}) \cap (\mathit{Pos} \to \mathit{Pos})))\ 0$
- $\longrightarrow \flat_0 \blacktriangleleft (\flat_1 \cap \flat_2) : \cdots,$
  $(((\lambda x.x + 1)\ @^{\flat_1}\ (\mathit{Even} \to \mathit{Even}))\ @^{\flat_2}\ (\mathit{Pos} \to \mathit{Pos}))\ 0$
- $\longrightarrow \flat_2 \blacktriangleleft (\flat_3 \to \flat_4) : \cdots,$
  $(((\lambda x.x + 1)\ @^{\flat_1}\ (\mathit{Even} \to \mathit{Even}))\ (0\ @^{\flat_3}\ \mathit{Pos}))\ @^{\flat_4}\ \mathit{Pos}$
- $\longrightarrow \flat_3 \blacktriangleleft (\mathit{false}) : \cdots,$
  $(((\lambda x.x + 1)\ @^{\flat_1}\ (\mathit{Even} \to \mathit{Even}))\ 0)\ @^{\flat_4}\ \mathit{Pos}$
- $\longrightarrow \flat_1 \blacktriangleleft (\flat_5 \to \flat_6) : \cdots,$
  $(((\lambda x.x + 1)\ (0\ @^{\flat_5}\ \mathit{Even}))\ @^{\flat_6}\ \mathit{Even})\ @^{\flat_4}\ \mathit{Pos}$
- $\longrightarrow \flat_5 \blacktriangleleft (\mathit{true}) : \cdots,$
  $(((\lambda x.x + 1)\ 0)\ @\ \mathit{Even})\ @\ \mathit{Pos}$
- $\longrightarrow \cdots,$
  $(1\ @^{\flat_6}\ \mathit{Even})\ @^{\flat_4}\ \mathit{Pos}$
- $\longrightarrow \flat_6 \blacktriangleleft (\mathit{false}) : \cdots,$
  $\mathit{blame}^{\flat}$

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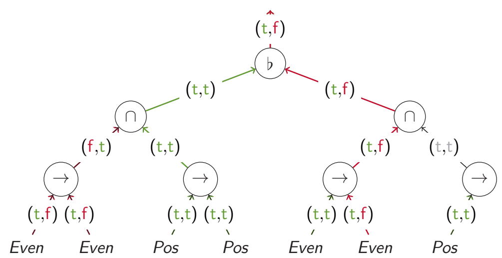
Constraint Graph



Example

- $((\lambda x.x + 1)\ @^{\flat}\ ((\mathit{Even} \to \mathit{Even}) \cap (\mathit{Pos} \to \mathit{Pos})))\ 1 \longrightarrow^{*} 2$ ✓
- $((\lambda x.x + 1)\ @^{\flat}\ ((\mathit{Even} \to \mathit{Even}) \cap (\mathit{Pos} \to \mathit{Pos})))\ 2 \longrightarrow^{*}$ ✗

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Technical Results

Definition (Contract Satisfaction)

The semantics of a contract $\mathcal{C}$ defines

1. a set $[\![\mathcal{C}]\!]^+$ of *closed terms* (subjects) that *satisfy* $\mathcal{C}$
2. a set $[\![\mathcal{C}]\!]^-$ of *closed contexts* that *respect* $\mathcal{C}$

The definition is mutually inductive on the structure of $\mathcal{C}$.

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Technical Results (cont'd)

Theorem (Contract soundness for expressions)

For all $M, \mathcal{C}, \flat.\ M @^\flat \mathcal{C} \in [\![\mathcal{C}]\!]^+$

Theorem (Contract soundness for contexts)

For all $\mathcal{L}, \mathcal{C}, \flat.\ \mathcal{L}[\square @^\flat \mathcal{C}] \in [\![\mathcal{C}]\!]^-$

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Technical Results (cont'd)

Theorem (Subject blame soundness)

Suppose that $M \in [\![\mathcal{C}]\!]^+$.
If $\varsigma, E[M @^\flat \mathcal{C}] \longmapsto^* \varsigma', N$ *then* $[\![\varsigma']\!](\flat, \mathit{subject}) \sqsubseteq \mathsf{t}$.

Theorem (Context blame soundness)

Suppose that $\mathcal{L} \in [\![\mathcal{C}]\!]^-$.
If $\varsigma, \mathcal{L}[M @^\flat \mathcal{C}] \longmapsto^* \varsigma', N$, *then* $[\![\varsigma']\!](\flat, \mathit{context}) \sqsubseteq \mathsf{t}$.

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